Scheme for generating the cluster states via atomic ensembles

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A scheme for generating the cluster states via atomic ensembles is proposed. The scheme has inherent fault tolerance function and is robust to realistic noise and imperfections. All the facilities used in our scheme are well within the current technology.

PACS numbers: 03.67.Hk, 03.65.Ud, 42.50.Dv

Entanglement is one of the most counterintuitive features in quantum mechanics. One can realize many impossible tasks within the classical world with the help of entangled states. Some striking applications of entanglement including quantum dense coding [1], quantum teleportation [2] and quantum cryptography [3] have been proposed. While bipartite entangled state is well understood, multipartite entanglement is still under extensive exploration. People soon realized that it isn't just an extension of bipartite entanglement. It not only inviolates local realism in a much stronger way, but also is a useful resource in quantum information processing. For tripartite entangled quantum system, it falls into two classes of irreducible entanglement [4, 5, 6]. Now great efforts are engaged in the investigation of multipartite entanglement with its promising features, such as, decoherence-free quantum information processing, multiparty quantum communications and so on. Recently, Briegel *et al.* [7] introduced a class of N-qubit entangled states, *i.e.*, the cluster states, which share the entanglement properties both in the GHZ- and W-class entangled states. But they still have some different properties, *e.g.*, they (in the case of N¿4) are harder to be destroyed by local operations than GHZ-class states. In addition, they are also an universal resource for quantum computation [8].

Due to its promising applications in quantum information processing and quantum computation, the cluster states have attracted much attention. Recently Zou *et al.* proposed probabilistic schemes for generating the entangled cluster states of four distant trapped atoms in leaky cavities [9], of atoms in resonant microwave cavities [10] and of four-photon polarization via linear optics [11]. Meanwhile, many researchers pay their attentions to atomic ensembles in realizing the scalable long-distance quantum communication [12]. The schemes based on atomic ensembles have some peculiar advantages compared with the schemes of quantum information processing by the control of single particles. Firstly, the schemes have inherent fault tolerance function and are robust to realistic noise and imperfections. Laser manipulation of atomic ensembles without separately addressing the individual atoms is dominantly easier than the coherent control of single particles. In addition, atomic ensembles with suitable level structure could have some kinds of collectively enhanced coupling to certain optical mode due to the multi-atom interference effects. Due to the above distinct advantages, a lot of novel schemes for the generation of quantum entangled states and quantum information processing have been proposed by using atomic ensembles [13, 14, 15, 16, 17].

Here, we suggest a scheme to generate the cluster states via atomic ensembles with a large number of identical alkali metal atoms as basic system. The relevant level structure of the alkali metal atoms is shown in Fig. 1. $|g\rangle$ is the ground state, $|e\rangle$ is the excited state and $|h\rangle$, $|v\rangle$ are two metastable states for storing a qubit of information, *e.g.*, Zeeman or hyperfine sublevels. For the three levels $|g\rangle$, $|h\rangle$ and $|v\rangle$, which can be coupled via a Raman process, two collective atomic operators can be defined as

$$s = (1/\sqrt{N})\Sigma_{i=1}^{N_a}|g\rangle_i\langle s|,$$

where $s = h, v$, and $N_a \gg 1$ is the total number of atoms. s is similar to independent bosonic mode operators providing all the atoms remain in ground state $|g\rangle$. The states of the atomic ensemble can be express as $|s\rangle = s^+|vac\rangle$ $(s = h, v)$ after the emission of the single Stokes photon in a forward direction, where $|vac\rangle \equiv \otimes_{i=1}^{N_a}|g\rangle_i$ denotes the ground state of the atomic ensemble.

We firstly briefly discuss the generation of bipartite cluster states via atomic ensembles. The atomic ensembles 1 and 2 are both initially prepared in the state

$$|\phi\rangle_{12} = v_1^+ v_2^+|vac\rangle_{12} \tag{1}$$

using Raman pulses. All the single-qubit transformation can be achieved by laser pulses in atomic ensembles. Secondly, we perform a single-qubit operation on atomic ensemble 1

$$v_1^+|vac\rangle_1 \to (h_1^+ + v_1^+)|vac\rangle_1/\sqrt{2}. \tag{2}$$

Then, we perform a controlled-not transformation on the two atomic ensembles, where atomic ensemble 1 serving as controlled qubit and atomic ensemble 2 as target qubit. The scheme for implementing controlled-not gate via atomic ensembles with the help of Raman laser manipulations, beam splitters, and single-photon detections

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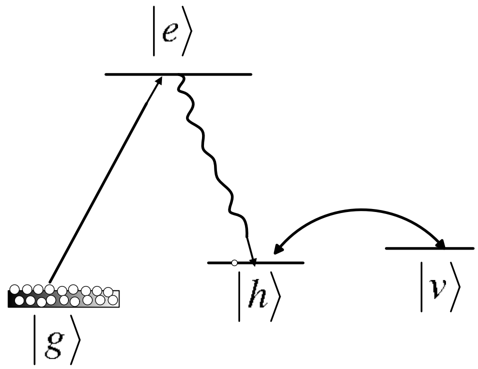

FIG. 1: The relevant atomic level structure of alkali metal atom. The transition of $|e\rangle \rightarrow |h\rangle$ can emit a forward-scattered Stokes photon that is co-propagating with the laser pulse. The excitation in the mode h can be transferred to optical excitation by applying an anti-pump pulse.

have been proposed in Ref. [18]. Now, the above procedures lead Eq. (1) to

$$|\phi\rangle_{12} = (h_1^+ v_2^+ + v_1^+ h_2^+)|vac\rangle_{12}/\sqrt{2}. \quad (3)$$

Finally, we perform a single-qubit operation on atomic ensemble 1

$$h_1^+|vac\rangle_1 \rightarrow v_1^+|vac\rangle_1, v_1^+|vac\rangle_1 \rightarrow h_1^+|vac\rangle_1, \quad (4)$$

and another single-qubit operation on atomic ensemble 2

$$\begin{aligned} h_2^+|vac\rangle_2 &\rightarrow (h_2^+ - v_2^+)|vac\rangle_2/\sqrt{2}, \\ v_2^+|vac\rangle_2 &\rightarrow (h_2^+ + v_2^+)|vac\rangle_2/\sqrt{2}. \end{aligned} \quad (5)$$

Here, the quantum state of atomic ensembles 1 and 2 becomes

$$\begin{aligned} |\phi\rangle_{12} &= [h_1^+(h_2^+ - v_2^+) + v_1^+(h_2^+ + v_2^+)]|vac\rangle_{12}/2 \\ &= [(h_1^+\sigma^2 + v_1^+)(h_2^+ + v_2^+)]|vac\rangle_{12}/2. \end{aligned} \quad (6)$$

Obviously the state is a standard bipartite cluster states ($N = 2$). The cluster states ($N = 2, 3$) can be also generated without controlled-not transformation [12, 13]. However, for the generation of the multipartite cluster states, using the proposals of Ref. [12, 13] are very hard, while by the above method with controlled-not transformations is simply and effective, as shown below. The multipartite cluster states are also very important in quantum information processing and quantum computation, so some of applications have been proposed [19, 20, 21, 22]. Thus the generation of the multipartite cluster states is necessary.

Out of question, for the generation of arbitrary N-particle cluster state ($N \geq 2$), we can perform the single-qubit operations and controlled-not transformations to achieve the task perfectly. Here, we discuss the process in detail. Firstly, we prepare N atomic ensembles, which are all in the states $v_i^+|vac\rangle_i$ ($i = 1, 2 \cdots N$). So the state of the whole system is

$$|\phi\rangle_{12\cdots N} = (v_1^+ v_2^+ \cdots v_N^+)|vac\rangle_{12\cdots N}. \quad (7)$$

Secondly, we perform appropriately transformations as the above process on atomic ensembles 1 and 2 (Eq.(2)-(6)), which lead the initial state to

$$\begin{aligned} |\phi\rangle_{12\cdots N} = (h_1^+\sigma^2 + v_1^+)(h_2^+ + v_2^+) \\ (v_3^+ v_4^+ \cdots v_N^+)|vac\rangle_{12\cdots N}/2. \end{aligned} \quad (8)$$

Then, we perform the same transformations on atomic ensembles 2 and 3 as atomic ensembles 1 and 2. We can obtain the result

$$\begin{aligned} |\phi\rangle_{12\cdots N} &= (h_1^+\sigma^2 + v_1^+)(h_2^+\sigma^3 + v_2^+) \\ & (h_3^+ + v_3^+)(v_4^+ v_5^+ \cdots v_N^+)|vac\rangle_{12\cdots N}/2\sqrt{2}. \end{aligned} \quad (9)$$

In a word, if we perform the transformations of Eq. (2)-(6) on atomic ensembles 1 and 2, then on atomic ensembles 2 and 3, up to on atomic ensembles $N-1$ and N, we will obtain the perfect multipartite cluster state

$$\begin{aligned} |\phi\rangle_{12\cdots N} &= \frac{1}{2^{N/2}}(h_1^+\sigma^2 + v_1^+)(h_2^+\sigma^3 + v_2^+) \\ &\cdots (h_N^+ + v_N^+)|vac\rangle_{12\cdots N} \\ &= \frac{1}{2^{N/2}} \otimes_{i=1}^N (h_i^+\sigma^{i+1} + v_i^+)|vac\rangle_i, \end{aligned} \quad (10)$$

where $\sigma^{N+1} \equiv 1$.

In summary, we have proposed a physical scheme to generate the cluster states, which are all maximally connected. The cluster states have some special features: it has a large persistency of entanglement; it can be regarded as a resource for other multi-qubit entangled states and so on. So the generation of cluster state is of great significance in the field of quantum information. Furthermore, the states have been applied to the quantum information processing and quantum computation [19, 20, 21, 22]. The cluster states have been generated in several systems, but the generation via atomic ensemble is still worthy consideration. Because the schemes based on atomic ensembles have some special advantages compared with others , *i.e.* the scheme of atomic ensemble has inherent fault tolerance function and are robust to realistic noise and imperfections. At the same time, the theory is simple and feasible. We can generate the N-qubit cluster state simply by extending the two-qubit case. Our scheme involves Raman-type laser manipulations, beam splitters, and single-photon detections, and the requirements of which are all well within the current experimental technology.

Acknowledgments

This work is supported by the Natural Science Foundation of the Education Department of Anhui Province under Grant No: 2004kj005zd and Anhui Provincial Natural Science Foundation under Grant No: 03042401 and the Talent Foundation of Anhui University.

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